Vector interpretation of one algebraic inequality.

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Let $a,b,c,d \in [2,4]$. Prove inequality $25(ab+cd)^2 \geq (a^2+d^2)(b^2+c^2)$.

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Let $\mathbf{x}(a,d)$ and $\mathbf{y}(b,c)$ be any two vectors such that $a,b,c,d \in [2,4]$ and let $\varphi(a,b,c,d)$ be angle between $\mathbf{x}$ and $\mathbf{y}$. Then $\cos\varphi(a,b,c,d) = \dfrac{\mathbf{x} \cdot \mathbf{y}}{\|\mathbf{x}\|\|\mathbf{y}\|} = \dfrac{ab+cd}{\sqrt{a^2+d^2} \cdot \sqrt{b^2+c^2}}$.

Noting that $\cos\varphi(a,b,c,d) > 0$ for any $a,b,c,d \in [2,4]$ we obtain

$\varphi(a,b,c,d) \leq \varphi(2,4,2,4) \Leftrightarrow \cos\varphi(a,b,c,d) \geq \cos\varphi(2,4,2,4) \Leftrightarrow$

$\dfrac{ab+cd}{\sqrt{a^2+d^2} \cdot \sqrt{b^2+c^2}} \geq \dfrac{2 \cdot 4 + 4 \cdot 2}{\sqrt{2^2+4^2} \cdot \sqrt{4^2+2^2}} = \dfrac{4}{5}$.

Hence, $\dfrac{(ab+cd)^2}{(a^2+d^2) \cdot (b^2+c^2)} \geq \dfrac{16}{25} \Leftrightarrow 25(ab+cd)^2 \geq 16(a^2+d^2)(b^2+c^2)$.